Lesson 3-3: Parallel Lines & Triangle Angle-Sum Theorem

Investigation

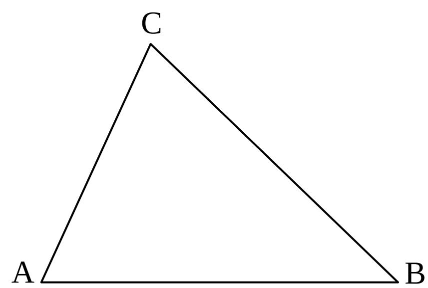
Take a look at the Investigation on page 131 of the text. Follow it to make a conjecture about the sum of the angle measures of a triangle. Try it with a few different triangles.

No matter what triangle you construct, you will find that the angles combine to form a straight angle (measure equals 180). From this we can form the following conjecture:

> The sum of the measures of the angles of a triangle is 180.

As with any conjecture that is true in a few specific situations, we need to prove it works in all situations. To do so we need to draw a diagram and get creative. This is a tough proof until you can see the "trick."

Planning the proof

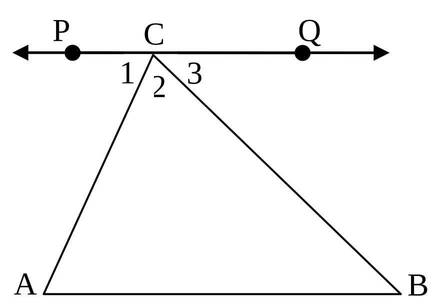


The first step, as always, is to determine which tricks in our toolbox (postulates, theorems) can be used to help. Thus far, almost all of our postulates and theorems deal with pairs of angles and parallel lines. Looking at the diagram, we have three angles (A, B and C) and three line segments ($\overline{AB}$, $\overline{BC}$ and $\overline{AC}$). Not a lot to work with is it?

What if we added a parallel line to the diagram? Then we could try to identify angle pairs and apply our postulates and theorems. Pick a side: I'll pick $\overline{AB}$. I want a line parallel to $\overline{AB}$ that will form interesting angle pairs. Problem is how do we construct a line we know is parallel to $\overline{AB}$? Well, we have Theorem 3-3 which says if we have congruent alternate interior angles, the transversed lines are parallel. Take angle A. Let's construct ray PC so that $\angle A \cong \angle PCA$; this will guarantee ray PC is parallel to $\overline{AB}$. Extend ray PC in the opposite direction to form line PC which is parallel to $\overline{AB}$ (place pt. Q on the opposite side). This will allow us to conclude that $\angle B \cong \angle QCB$ because of Theorem 3-1 (Alternate Interior Angles Postulate).

Here is the new diagram with $\angle PCA$ renamed as $\angle 1$, $\angle QCB$ renamed as $\angle 3$ and $\angle C$ renamed as $\angle 2$.

Proof



Line $PQ \parallel \overline{AB}$	Given
$\angle A \cong \angle 1$	Alt int $\angle$'s (Thm 3-1)
$\angle B \cong \angle 3$	Alt int $\angle$'s (Thm 3-1)
$m\angle 1 + m\angle 3 + m\angle 2 = 180$	Angle Add Post.
$m\angle A + m\angle B + m\angle C = 180$	Subst POE
Q.E.D.	

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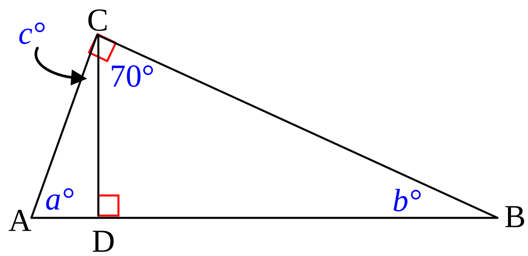
Theorem 3-7 Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180.

Putting it to work…the algebra connection again

Consider the following diagram.
Given: $\angle ACB$ is a right angle
$\overline{CD} \perp \overline{AB}$
Find the values of *a*, *b* and *c*.



$c = 90 - 70 = 20$ (complementary $\angle$'s)

$a + c + 90 = 180$ (Theorem 3-7)
$a = 180 - 90 - c = 90 - 20 = 70$

$b + 70 + 90 = 180$ (Theorem 3-7)
$b = 180 - 160 = 20$

Classifying triangles

Just as we classify angles, we classify triangles. We classify them with a combination of angle and side relationships. Look at the top of pg 133 for diagram examples:

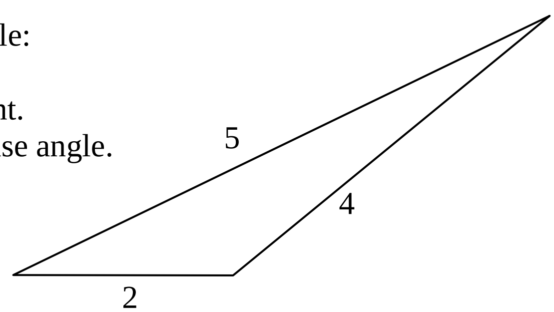
By angle:

- Equiangular: all angles are congruent
- Acute: all angles are acute (measure is greater than 0 and less than 90)
- Right: one angle is a right angle
- Obtuse: one angle is obtuse (measure is greater than 90 and less than 180)

…and in combination with side:

- Equilateral: all sides are congruent
- Isosceles: two sides are congruent
- Scalene: none of the sides are congruent

For an example, classify the following triangle:



Consider the sides: none are congruent.
Consider the angles: there is one obtuse angle.

This is an obtuse scalene triangle.

Going deeper…

Would it be possible to have an obtuse equiangular triangle?

Take a look at the classification; we classify by a combination of angle & side relationships. What relationships are being used in this classification? Hmm, both are angle relationships. That ain't good. So, nope, we can't get there from here. No, goal, no score, can't do it.

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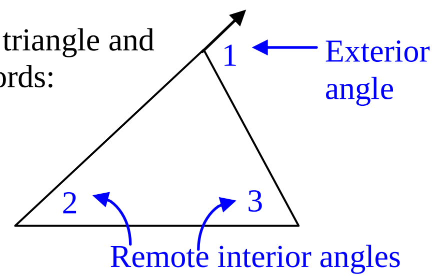
OK, what about an obtuse equilateral triangle? Well, we have both side and angle relationships, so we are ok there. What do you think?

Consider an equilateral triangle. Construct a few. It will quickly become evident to you that for all sides to be congruent, all angles must be acute. Hence, you can't have an obtuse equilateral triangle.

Exterior angles

In Theorem 3-7 we were considering the interior angles of the triangle. What can we discover about exterior angles of triangles? First, we should define what we mean by an exterior angle of a triangle.

An exterior angle of a triangle is the angle formed by a side of the triangle and an extension of an adjacent side. A picture is worth a thousand words:



Remote interior angles

The two interior angles of the triangle at the other vertices are called remote interior angles.

Great! We have two new angles. Can you guess what I'm going to ask you now? Yup! Let's make a conjecture about how an exterior angle relates to its remote interior angles.

Draw and cut out a triangle. Number the interior angles. Tear off angles 2 and 3 leaving most of the triangle in place. Arrange these two angle pieces over the exterior angle at angle 1. Notice anything? They look like together they are congruent with the exterior angle. And indeed they are!

Theorem 3-8 Triangle Exterior Angle Theorem

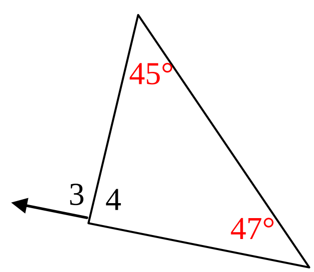
The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles. Using the diagram above:

$$m\angle 1 = m\angle 2 + m\angle 3$$

What's that? The proof? Great idea! I'll leave it for you in problem #49. ☺

Algebra and Theorem 3-8 at work

Consider problem #28 pg 135. Find each missing angle measure.



$m\angle 3 = 45 + 47 = 92$ (Theorem 3-8)
$m\angle 3 + m\angle 4 = 180$ (Supplementary angles)
$m\angle 4 = 180 - m\angle 3 = 180 - 92 = 88$ (Subst POE)

Assign homework

p. 134 #1-11, 17-33 odd, 43-49 odd, 52, 64-67
p. 139 #1-10